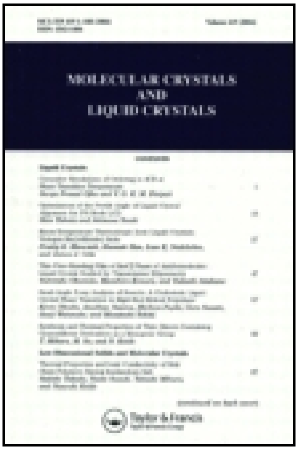

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Pattern Formation in Thermal Convective Nematic Liquid Crystalsi

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Pattern Formation in Thermal Convective Nematic Liquid Crystals†

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The amplitude equation for thermal convective nematic liquid crystals is derived from the equations of motion. The appearance of various patterns (normal rolls, undulations, zigzags, etc.) in different parameter regimes is described in a unified manner. The effect of a horizontal magnetic field is included and the corresponding phase diffusion equation is given.

Keywords: liquid crystal, pattern formation, amplitude equation, thermal convection, Rayleigh-Benard, zigzag

I. INTRODUCTION

For simple liquids undergoing thermal convections, the Rayleigh-Benard and other instabilities are well studied, especially by the use of amplitude equations[1,2] and diffusion equations.[3] The resulting theoretical "phase diagram" compares reasonably well with the experiments.

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On the other hand, anisotropic liquids (such as liquid crystals) are much more complicated and are less studied as nonlinear instabilities are concerned. For nematics, linear stability analysis of the first threshold (of parallel rolls) has been carried out[4] and is understood. Yet, the study of nonlinear instabilities is still in its early stage.[5,6] Recently, new experiments in thermal convective nematics (including those with a vertical magnetic field[7] or a horizontal magnetic field[8]) are reported. To understand these experimental results detailed theoretical treatment of the nonlinear terms is needed.

In this paper we start from the nonlinear equations of motion of nematics under thermal gradient and in the presence of a magnetic field and *derive* the corresponding amplitude equation using multiple scales analysis[1] (see Sec. II). The diffusion equation of phase describing transverse undulations and longitudinal perturbations is obtained in Sec. III. Various patterns are then given. The interesting result is that our amplitude equation has the same form as that in the simple (isotropic) liquid case. The important difference is that the four coefficients in our case can be changed (to zero or in sign depending on the material parameters and the magnetic field) while in the simple liquid case they are fixed (see Sec. IV). Consequently, a rich variety of physical results are possible in liquid crystals. Sec. V concludes the paper.

II. THE AMPLITUDE EQUATION

A. Basic equations of motion

A planar nematic liquid crystal cell of thickness d is under a thermal gradient with the two plates maintained at different temperatures (Figure 1). Let the temperature distribution be $T = T_0 - \beta z + u(x, y, z, t)$ where β is a constant, $T_0 - \beta z$ is the external temperature field and $u(x, y, z, t)$ is the perturbation due to the motion of the liquid crystal molecules. In the presence of an external magnetic field $\mathbf{H} = (H_x, H_y, H_z)$ the equations of motion of the nematic is given by[9]

$$\begin{aligned} &\nabla \cdot \mathbf{v} = 0 \\ &\rho \partial \mathbf{v}/\partial t + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = \rho g \alpha u \hat{z} + \nabla \cdot \overleftrightarrow{t} \\ &\mathbf{\Gamma}_e + \mathbf{\Gamma}_v = 0 \\ &\partial T/\partial t + (\mathbf{v}\cdot\nabla)T = K_\perp \nabla^2 T + K_a \nabla\cdot[\mathbf{n}(\mathbf{n}\cdot\nabla T)] \end{aligned} \qquad (2.1)$$

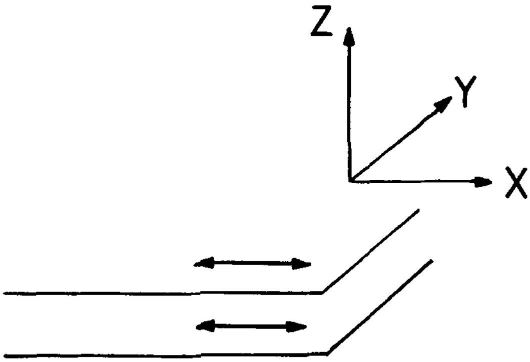

FIGURE 1 The planar liquid crystal cell. Molecules are along x axis at the two surfaces at $z = \pm d/2$.

where K_a is the anisotropy of thermal conductivity and χ_a the anisotropy of magnetic susceptibility, with

$$t_{ij} \equiv -p\delta_{ij} - (\partial F/\partial n_{l,j})n_{l,j} + t'_{ij}$$

$$\overleftrightarrow{t}' \equiv \alpha_1 \mathbf{nn}(\mathbf{n}\cdot\overleftrightarrow{A}\cdot\mathbf{n}) + \alpha_2\mathbf{nN} + \alpha_3\mathbf{Nn} + \alpha_4\overleftrightarrow{A}$$

$$+ \alpha_5\,\mathbf{n}(\mathbf{n}\cdot\overleftrightarrow{A}) + \alpha_6\,(\mathbf{n}\cdot\overleftrightarrow{A})\mathbf{n}$$

$$A_{ij} \equiv \frac{1}{2}(v_{i,j} + v_{j,i}),\ \mathbf{N} \equiv d\mathbf{n}/dt - \frac{1}{2}(\nabla\times\mathbf{v})\times\mathbf{n}$$

$$F = \frac{1}{2}kn_{i,j}n_{i,j} - \frac{1}{2}\chi_a(n_iH_i)^2,\ f_{i,j} \equiv \partial f_i/\partial x_j,$$

$$\Gamma_e \equiv -\mathbf{n}\times(\delta F/\delta\mathbf{n}),\ \Gamma_v \equiv \mathbf{n}\times(\gamma_1\mathbf{N} + \gamma_2\overleftrightarrow{A}\cdot\mathbf{n})$$

$$K_a \equiv K_\parallel - K_\perp$$

Here, $\mathbf{v} = (v_x, v_y, v_z)$ is the velocity of the center of mass of the molecule, $\mathbf{n} = (n_x, n_y, n_z)$ the director, ρ the density, g the acceleration due to gravity, α the bulk thermal expansion coefficient, p the pressure, k the Frank elastic constant (one-constant approximation is assumed).

B. Physical approximations

Let $n_x = \cos\phi\cos\delta$, $n_y = \sin\delta$, $n_z = \sin\phi\cos\delta$ (see Figure 2) where

$$\phi = \phi(x, y, z, t), \qquad \delta = \delta(x, y, z, t) \tag{2.2}$$

In other words, δ is the deviation angle of **n** with respect to the (x, z) plane; ϕ is the angle between the x axis and the projection of **n** on the (x, z) plane.

For simplicity, we assume $\phi, \delta << 1$ (see Sec. V) and take

$$\sin\phi \simeq \phi,\ \cos\phi \simeq 1,\ \sin\delta \simeq \delta,\ \cos\delta \simeq 1 \tag{2.3}$$

Using (2.2) and (2.3), (2.1) becomes

$$\begin{aligned}
& v_{x,x} + v_{y,y} + v_{z,z} = 0 \\
& \Delta_K u + K_a\beta\phi,_x - \beta v_z = N_2 \\
& -\Delta_\delta\delta + \alpha_3 v_{x,y} + \alpha_2 v_{y,x} = N_3 \\
& -\Delta_\phi\phi + \alpha_3 v_{x,z} + \alpha_2 v_{z,x} = N_4 \\
& -p_{,x} + \alpha_3\phi,_{tz} + \alpha_3\delta,_{ty} + \Delta_1 v_x + \frac{1}{2}\alpha_{245} v_{y,xy} + \frac{1}{2}\alpha_{245} v_{z,xz} = N_5 \qquad (2.4) \\
& -p_{,y} + \alpha_2\delta,_{tx} + \frac{1}{2}\alpha_{245} V_{x,xy} + \Delta_2 v_y + \frac{1}{2}\alpha_4 v_{z,yz} = N_6 \\
& -p_{,z} + \rho g\alpha u + \alpha_2\phi,_{xt} + \frac{1}{2}\alpha_{245} v_{x,xz} + \frac{1}{2}\alpha_4 v_{y,yz} + \Delta_3 v_z = N_7
\end{aligned}$$

where

$$\Delta_K \equiv \frac{\partial}{\partial t} - \left(K_\parallel \frac{\partial^2}{\partial x^2} + K_\perp \frac{\partial^2}{\partial y^2} + K_\perp \frac{\partial^2}{\partial z^2}\right)$$

$$\Delta_\phi = \Delta_\delta \equiv -\gamma_1 \frac{d}{dt} - k\Delta + \chi_a H_x^2$$

$$\Delta_1 \equiv \alpha_{456} \frac{\partial^2}{\partial x^2} + \frac{1}{2}\alpha_{346} \frac{\partial^2}{\partial y^2} + \frac{1}{2}\alpha_{346} \frac{\partial^2}{\partial z^2}$$

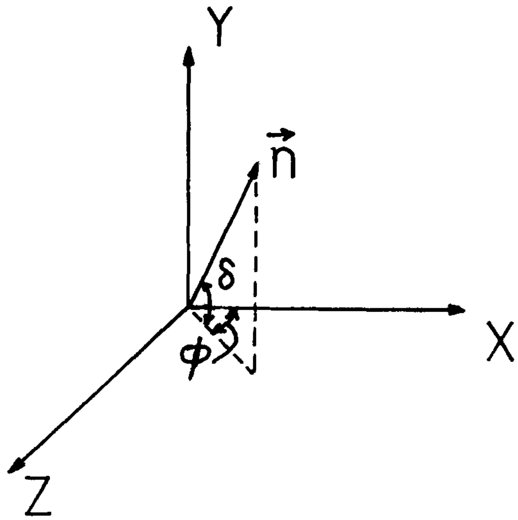

FIGURE 2 Definitions of ϕ and δ of the director **n**.

$$\Delta_2 \equiv \frac{1}{2}\alpha_{\bar{2}45}\frac{\partial^2}{\partial x^2} + \alpha_4\frac{\partial^2}{\partial y^2} + \frac{1}{2}\alpha_4\frac{\partial^2}{\partial z^2} \tag{2.5}$$

$$\Delta_3 \equiv \frac{1}{2}\alpha_{\bar{2}45}\frac{\partial^2}{\partial x^2} + \frac{1}{2}\alpha_4\frac{\partial^2}{\partial y^2} + \alpha_4\frac{\partial^2}{\partial z^2}$$

$$\alpha_{ijk\ldots} \equiv \alpha_i + \alpha_j + \alpha_k + \ldots$$

$$\alpha_{\bar{i}jk\ldots} \equiv -\alpha_i + \alpha_j + \alpha_k + \ldots$$

For simplicity, in (2.4) and (2.5) we have assumed $\mathbf{H} = (H_x, O, O)$ and $\alpha_1 \simeq O$. The latter is justified in many nematic materials (e.g. MBBA) in which α_1 is usually quite small. In (2.4) the expressions for N_i $(i = 2 - 7)$ (and many other quantities below) are quite complicated and too lengthy to be reproduced here (which are available from the authors upon request). For our purposes here it is sufficient to note that $N_i = N_i\,(v_x, v_y, v_z, \phi, \delta, u, p)$.

The boundary conditions for (2.4) are[5]

$$v_x = v_y = \delta = \phi = 0 \text{ at } z = \pm d/2$$

$$\partial v_z/\partial z = \partial\phi/\partial z = \partial u/\partial z = 0 \text{ at } z = \pm d/2 \tag{2.6}$$

in which the free-free boundary condition is adopted.

Note that (2.4) is reduced to the linear equations[4,10] if we let the nonlinear terms $N_i = 0$ and $\delta = 0$. The nonlinear case of $\delta = 0$ and $H = 0$ has been discussed in Ref. 5.

C. The Single equation of v_z

Eq. (2.4) is the simultaneous equation of seven independent variables $(v_x, v_y, v_z, \delta, \phi, u, p) = f$. By simple algebra one can obtain a single equation with only v_z in the linear part,

$$Lv_z = N \tag{2.7}$$

where[11]

$$\begin{aligned} L &\equiv \frac{\partial}{\partial x}(\Delta_5\Delta_7 - \Delta_8\Delta_9) \\ N &\equiv \Delta_7 N_{15} - \Delta_9 N_{16} = N(f) \end{aligned} \tag{2.8}$$

Note that the material anisotropy shows up in the anisotropy of the viscosities (α_i), the thermal conductivities ($K_\parallel$, $K_\perp$), the magnetic susceptibility (χ_a) (whereas the elastic anisotropy has been approximated to be zero already in (2.1)). Therefore, in (2.7), if one lets $\alpha_2 = \alpha_3 = \alpha_5 = \alpha_6 = 0$, $\frac{1}{2}\alpha_4 = \nu$, $K_a = 0$, $\chi_a = 0$ (equivalent to $H = 0$ here) one should get back the case of isotropic simple liquid.[1] Indeed, (2.7) is found to be reduced to

$$\begin{aligned} &L_1\left[-\rho g\alpha\beta\nabla_1^2 + \left(\frac{\partial}{\partial t} - k\Delta\right)\left(\rho\frac{\partial}{\partial t} - \nu\Delta\right)\Delta\right]v_z \\ &= L_1\left[-\rho g\alpha\nabla_1^2(\mathbf{v}\cdot\nabla T) + \left(\frac{\partial}{\partial t} - k\Delta\right)(\nabla\times\nabla\times(\mathbf{\Omega}\times\mathbf{v})\cdot\hat{z})\right] \end{aligned} \tag{2.9}$$

where

$$\begin{aligned} &L_1 = k^2\frac{\partial}{\partial x}\left[\Delta^2\left(\nu\Delta - \rho\frac{\partial}{\partial t}\right)\right], \quad \nabla_1 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right), \\ &\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \end{aligned} \tag{2.10}$$

When the operator L_1 is removed (2.9) is exactly the Eq. (2.10) of Ref. 1.

D. The amplitude equation

Following Ref. 1 we let $\beta = \beta_0 + \epsilon^2\beta_2$ and introduce the slow variables

$$X = \epsilon x,\ Y = \epsilon^{1/2}y,\ \mathrm{T} = \epsilon^2 t,\ \epsilon << 1 \tag{2.11}$$

(Here T is time and *not* temperature.) In (2.4) use

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial x} + \epsilon\frac{\partial}{\partial X}$$

$$\frac{\partial}{\partial y} \to \frac{\partial}{\partial y} + \epsilon^{\frac{1}{2}}\frac{\partial}{\partial Y}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \epsilon^2\frac{\partial}{\partial \mathrm{T}} \tag{2.12}$$

and expand the variables in a series,

$$f = \epsilon f^{(1)} + \epsilon^{3/2}f^{(3/2)} + \epsilon^2 f^{(2)} + \ldots \tag{2.13}$$

Substituting (2.11)–(2.13) into (2.4) one can solve for $f^{(1)}, f^{(3/2)}, f^{(2)}, f^{(5/2)}$, etc. In particular, $f^{(1)}$ corresponds to the linear case and possesses the solution,

$$v_x^{(1)} = \mathrm{AC}_-,\ v_y^{(1)} = 0,\ v_z^{(1)} = \mathrm{S}_+,\ \phi^{(1)} = \mathrm{CS}_-,\ \delta^{(1)} = 0,$$

$$p^{(1)} = \mathrm{DC}_+,\ u^{(1)} = \mathrm{BS}_+ \tag{2.14}$$

where

$$\mathrm{C}_\pm = [W(X, Y, \mathrm{T})e^{iqx} \pm \text{c.c.})]\cos\frac{\pi}{d}z$$

$$\mathrm{S}_\pm = [W(X, Y, \mathrm{T})e^{iqx} \pm \text{c.c.})]\sin\frac{\pi}{d}z \tag{2.15}$$

and

$$A = \frac{i\pi}{dq}$$

$$B = \beta_o \frac{[k(q^2 + \pi^2/d^2) + \chi_a H^2] - K_a(-\alpha_3\pi^2/d^2 + \alpha_2 q^2)}{(K_\| q^2 + K_\perp \pi^2/d^2)[k(q^2 + \pi^2/d^2) + \chi_a H^2]}$$

$$C = -i\frac{\alpha_2 q^2 - \alpha_3\pi^2/d^2}{[k(q^2+\pi^2/d^2)+\chi_a H^2]q}$$

$$D = -\frac{1}{2}\alpha_{34556}\,\pi/d - \frac{1}{2}\alpha_{346}\,\pi^3/(d^3q^2) \tag{2.16}$$

$f^{(3/2)}$, $f^{(2)}$ and $f^{(5/2)}$ can also be solved. In particular,

$$v_z^{(5/2)} = v_z^{(3/2)} = v_z^{(2)} = 0 \tag{2.17}$$

By (2.13) the lowest order of $N^{(i)}$ in (2.7) is $i = 2$, i.e., $N^{(1)} = N^{(3/2)} = 0$. Putting (2.12) and (2.13) into (2.7) we have for the first order,

$$L^{(0)}v_z^{(1)} = 0 \tag{2.18}$$

and for the third order,

$$L^{(2)}v_z^{(1)} + L^{(0)}v_z^{(3)} = N^{(3)} \tag{2.19}$$

Multiplying both sides of (2.19) from the left by $v_z^{(1)}$ we have

$$\langle v_z^{(1)}L^{(2)}v_z^{(1)}\rangle + \langle v_z^{(1)}\,L^{(0)}v_z^{(3)}\rangle = \langle v_z^{(1)}N^{(3)}\rangle \tag{2.20}$$

where

$$\langle fg\rangle \equiv \int_{-\infty}^{\infty} dx \int_{-d/2}^{d/2} dz f^* g \tag{2.21}$$

From the definition of $L^{(i)}$, (2.18) and the boundary condition (2.6) and using partial differentiation we have

$$\langle v_z^{(1)}L^{(0)}v_z^{(3)}\rangle = \langle v_z^{(3)}L^{(0)}v_z^{(1)}\rangle = 0 \tag{2.22}$$

Eq. (2.20) becomes

$$\langle v_z^{(1)}L^{(2)}v_z^{(1)}\rangle = \langle v_z^{(1)}N^{(3)}\rangle \tag{2.23}$$

From (2.13) it can be easily seen that among the variables contained in $N^{(3)}$ the highest order is $f^{(2)}$. Hence, in (2.23) there is no $f^{(3)}$ terms. In particular, there is no $v_z^{(3)}$ (and no $f^{(5/2)}$).

Putting (2.14)–(2.17) and the other solutions of $f^{(3/2)}$ and $f^{(2)}$ into (2.23) we obtain the amplitude equation.

$$\mu_4 \frac{\partial W}{\partial \mathrm{T}} + \mu_1 \frac{\partial^2 W}{\partial X^2} + i\mu_2 \frac{\partial^3 W}{\partial X \partial Y^2} + \mu_3 \frac{\partial^4 W}{\partial Y^4} = (\mu_5 + \mu_6 W W^*) W \tag{2.24}$$

Here, μ_i ($i = 1 - 6$) are coefficients depending on the material parameters and H (as well as q).

(When the anisotropy parts in μ_i are put to zero (2.24) reduces to the case of simple isotropic liquid and indeed becomes identical to Eq. (2.19) of Ref. 1.)

When (2.24) is divided by a common nonzero constant and T, X, Y, and W are rescaled and redefined five of the six coefficients in (2.24) can be made to have absolute magnitude of unity. In other words, (2.24) can be renormalized to be

$$\frac{\partial W}{\partial T} - a \frac{\partial^2 W}{\partial X^2} + i\mu \frac{\partial^3 W}{\partial X \partial Y^2} + c \frac{\partial^4 W}{\partial Y^4} = (1 - bWW^*) W \tag{2.25}$$

The reasoning goes something like this. Without loss of generality one can assume $\mu_4 > 0$. (If not, one can always multiply both sides of (2.24) by -1 to make $\mu_4 > 0$.) For $\beta > \beta_c$ $(=\beta_0)$, $\mu_5 > 0$.[5] We then let

$$\begin{gathered}
\sqrt{\frac{|\mu_6|}{\mu_5}}\, W \rightarrow W \\
\frac{\mu_4}{\sqrt{|\mu_6|\,\mu_5}} \frac{1}{\mathrm{T}} \rightarrow \frac{1}{\mathrm{T}} \\
\frac{-\mu_1}{\sqrt{|\mu_6|\,\mu_5}} \frac{1}{X^2} \rightarrow \frac{1}{X^2} \\
\frac{|\mu_3|}{\sqrt{|\mu_6|\,\mu_5}} \frac{1}{Y^4} \rightarrow \frac{1}{Y^4} \\
\frac{\mu_2}{\sqrt{\mu_1 \mu_3}} \rightarrow \mu
\end{gathered} \tag{2.26}$$

By redefining X one can always make $\mu \geqslant 0$. Putting (2.26) into (2.24) it then becomes clear that a, b, c can assume the values $+1$, -1 and 0 only.

The simple liquid case corresponds to $a = b = c = 1$ and $\mu = 2$. In this case the three spatial gradient terms can be combined to be

$$\left(\frac{\partial}{\partial X} - i\frac{\partial^2}{\partial Y^2}\right)^2.$$

Eq. (2.25) has a steady solution which is independent of Y,

$$W_S = [(1-aQ^2)/b]^{\frac{1}{2}}\exp(iQX) \tag{2.27}$$

III. THE PHASE DIFFUSION EQUATION

Let us consider a perturbed solution of (2.25) of the form[2]

$$W = W_s(1+u)\exp(i\phi) \tag{3.1}$$

(Here, u and ϕ are new variables not the same as in previous Sections.) Putting (2.27) into (3.1) and then (2.25), separating the real and imaginary parts and ignoring nonlinear terms in u and ϕ one obtains

$$\frac{\partial u}{\partial T} - a\frac{\partial^2 u}{\partial X^2} - \mu Q\frac{\partial^2 u}{\partial Y^2} + 2aQ\frac{\partial \phi}{\partial X} - \mu\frac{\partial^3 \phi}{\partial X \partial Y^2} + c\frac{\partial^4 u}{\partial Y^4} + 2u\,(1 - aQ^2) = 0 \tag{3.2}$$

$$\frac{\partial \phi}{\partial T} - a\frac{\partial^2 \phi}{\partial X^2} - \mu Q\frac{\partial^2 \phi}{\partial Y^2} - 2aQ\frac{\partial u}{\partial X} + \mu\frac{\partial^3 u}{\partial X \partial Y^2} + c\frac{\partial^4 \phi}{\partial Y^4} = 0 \tag{3.3}$$

Let $u \sim \exp(iK_xX+iK_yY)$. In the long-wavelength limit, K_x, $K_y \to 0$,[2] and assuming that the magnitude of W relaxes faster than the phase we then drop $\partial u/\partial T$ and the drivatives of u and (3.2) becomes

$$2aQ\frac{\partial\phi}{\partial X} - \mu\frac{\partial^3\phi}{\partial X\partial Y^2} + 2u(1-aQ^2) = 0 \tag{3.4}$$

Putting (3.4) into (3.3) and eliminating u we have

$$\frac{\partial\phi}{\partial T} = \frac{a(1-3aQ^2)}{1-aQ^2}\frac{\partial^2\phi}{\partial X^2} + \mu Q\frac{\partial^2\phi}{\partial Y^2} - c\frac{\partial^4\phi}{\partial Y^4} \tag{3.5}$$

which is the phase diffusion equation.

Note that (3.5) is independent of b. When $a = 1$, $b = c$, $\mu = 2$ (3.5) reduces to Eq. (32) of Ref. 2 (with $B_x = 0$ there), the result of simple liquids. Also, when $a = 1$ (3.5) has one more term (the last term on the rhs) than the usual phase diffusion equation (such as that in Ref. 2).

IV. STABILITY ANALYSIS

Let $\phi \sim \exp(\lambda T)\exp(iK_x'X + iK_y'Y)$ and put it into (3.5). The stability condition is given by

$$\lambda = -\frac{a(1-3aQ^2)}{1-aQ^2}K_x'^2 - \mu QK_y'^2 - cK_y'^4 < 0 \tag{4.1}$$

Let us consider two special cases.

A. When $K_y' \to 0$, the condition of Eckhaus instability is

$$Q^2 > 1/3a \tag{4.2}$$

B. $K_x' \to 0$. The condition corresponding to transverse (undulation) instability is

$$Q < -\mu^{-1}K_y'^2c \tag{4.3}$$

(Note that in Ref. 3, for example, the $cK_y'^4$ term in (4.1) is ignored in low-order calculations.)

By (2.24) and (2.26) we know that a, b, c and μ depend not only on the material parameters but also on the magnetic field. Consequently, by varying the magnetic field one can make the above coefficients change their values resulting in the change of stability conditions. The following are some interesting special cases.

1. When $a = b = c = 1$ and the magnetic field is varied, as long as a and c do not change (i.e. keeping $a = b = c = 1$) the Eckhaus instability condition (4.2) does not change. However, if a or c suddenly changes to zero when H (the magnetic field) is varied then the Eckhaus instability will disappear suddenly.

2. When $\mu \neq 0$ ($\mu > 0$) and H is continuously varied μ will also be changed. The instability region given by (4.3) will change accordingly. When μ approaches zero this instability region will vanish resulting in the disappearance of transverse instabilities.

3. When $b > 0$ one has direct bifurcation; for $b > 0$, indirect bifurcation (see, e.g., Ref. 5). Therefore, if b changes sign during the variation of H one will observe the sudden change of the bifurcation type (a "tricritical point").

It should be pointed out that the dependence of μ on H is in the following form:

$$\mu = (a_1 + b_1H^2 + c_1H^4)/|(a_2 + b_2H^2 + c_2H^4)(a_3 + b_3H^2 + c_3H^4)|^{1/2} \tag{4.4}$$

where a_i, b_i and c_i ($i = 1,2,3$) depend on the material parameters. Obviously,

$$\mu \sim c_1/|c_2c_3|^{1/2} \qquad \text{for } H >> 1 \tag{4.5}$$

and

$$\mu \sim a_1/|a_2a_3|^{1/2} \qquad \text{for } H << 1 \tag{4.6}$$

Denoting $a_i = a_{iI} + a_{iA}$, $i = 1,2,3$, where a_{iI} (a_{iA}) is the isotropic (anisotropic) part, it is obvious that one should have

$$a_{1I}|a_{2I}a_{3I}|^{-1/2} = 2 \tag{4.7}$$

V. CONCLUSION

We have presented our preliminary results of a theoretical investigation of thermal convective instabilities in an anisotropic liquid—nematic liquid crystal. It is in the planar configuration under a horizontal parallel magnetic field. At and beyond the parallel-roll threshold β_0 the amplitude equation is derived. This amplitude equation has similar form to that in the isotropic liquid case except that the four coefficients a, b, c and μ may assume different values ($a, b, c = \pm 1, 0, \mu \geq 0$) when the material parameters and the magnetic field are varied. Since the solution of the amplitude equation or the phase diffusion equation as well as the stability conditions and bifurcation types are related to the parameter regimes of a, b, c and μ one can expect to observe the change of stability regions and bifucation types when different nematics are used, or for a given nematic when the magnetic field is varied.

In this work, the vertical vorticity[2] has not been included. We cannot discuss skewed varicose (but normal rolls, undulation and zigzag are included). The approximation (2.3) has to be improved but it is not expected to change the form of the amplitude equation (2.5) except for the expressions of the coefficients. More detailed discussions, improvements and numerical results as well as comparision with experiments will be presented elsewhere.[11]

Note added in proof: In the electroconvective case amplitude equation similar to our Eq. (2.24) is obtained in Ref. 12. See also Ref. 13. We thank Dr. L. Kramer for calling our attention to Refs. 12 and 13 and valuable correspondence.

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